

**A COMPARISON BETWEEN IMSC, PI AND MIMSC METHODS
IN CONTROLLING THE VIBRATION OF FLEXIBLE SYSTEMS**

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August 1987

SUMMARY

This paper presents a comparative study between three active control algorithms which have proven to be successful in controlling the vibrations of large flexible systems. These algorithms are namely: the Independent Modal Space Control (IMSC), the Pseudo-inverse (PI) and the Modified Independent Modal Space Control (MIMSC).

Emphasis is placed, in this study, on demonstrating the effectiveness of the MIMSC method in controlling the vibration of large systems with small number of actuators by using an efficient time sharing strategy. Such a strategy favors the MIMSC over the IMSC method, which requires a large number of actuators to control equal number of modes, and also over the PI method which attempts to control large number of modes with smaller number of actuators through the use of an in-exact statistical realization of a modal controller.

Numerical examples are presented to illustrate the mains features of the three algorithms and the merits of the MIMSC method.

INTRODUCTION

Active control of the vibration of flexible systems has been recognized as essential to enhancing the stable operation of these systems when subjected to external disturbance. Strategies employed in the design of such control systems are based primarily on modal control methods whereby the flexible structures are controlled by controlling their dominant modes of vibrations. Generally, these modal control strategies belong to either the class of the coupled methods [1-6] or to the class of the independent modal space control (IMSC) method developed by Meirovitch and co-workers [7-12]. In the first class, although the open-loop equations of the system are uncoupled, the close-loop equations become coupled through the feedback controller. This makes the optimal computation of the feedback require the solution of a coupled matrix Riccati equation [3-6]. For large flexible structure the solution of the resulting Riccati equation can pose serious difficulties which limit significantly the applicability of the coupled modal control methods. The IMSC method avoids, however, such limitations as the control laws are designed completely, in the modal space maintaining the originally uncoupled open-loop equations of the system as a set of independent second-order equations even after including the modal feedback controllers. Meirovitch et al [7-12] showed, under such conditions, that it is possible to compute, in a close form, the optimal modal feedback gains. This feature makes the IMSC method computationally attractive and lends it suitable for controlling large structures.

The IMSC method requires, however, the use of as many actuators as

the number of modes to be controlled. Such a requirement results in practical limitation of the method when applied to large structures where the number of modeled modes can be very large.

Lindberg and Longman [13] proposed to modify the IMSC by using a small number of actuators to control all the modeled modes through the consideration of an approximate pseudo-inverse (PI) realization of the modal controller. This modification can result in physical control forces which can be far from desired because the PI is in effect a least square fit of N modal forces to obtain M physical forces. When $N=M$ then the realization is exact and is also the same as the IMSC. But as M becomes much smaller than N , i.e. when the number of actuators \ll number of modes, then the accuracy of the least square fit becomes increasingly poor. Accordingly, when the realized forces are transformed back to the modal space the resulting modal forces will be very far from the optimal forces and this will result in deterioration in the performance of the controller.

For these reasons, the Modified Independent Modal Space Control (MIMSC) method is initialed [14].

The MIMSC modifies the IMSC algorithm to account for the control spillover from the controlled modes into the uncontrolled modes when a small number of actuators is used to control a large number of modes. The method incorporates also an optimal placement procedure for determining the optimal location of the actuators in the structure. Moreover, the MIMSC method relies on an efficient algorithm for "Time Sharing" a small

number of actuators in the modal space to control a large number of modes. In effect the MIMSC uses M optimally placed actuators to control the M modes that have the highest modal energy at any instant of time and time share these actuators among the other residual modes when the control spillover makes their modal energy higher than the controlled modes.

Comparisons between the IMSC, PI and MIMSC algorithms are presented here to illustrate the main features and merits of these methods as applied to the control of vibrations of spring-mass systems and simple cantilever beams.

DESCRIPTION OF METHODS

Complex flexible systems can be modeled dynamically by a discrete finite element model as follows :

$$M\ddot{\delta} + K\delta = F \quad (1)$$

where M is the overall mass matrix of the structure

K is the overall stiffness matrix of the structure

δ and $\ddot{\delta}$ are the displacement and acceleration of the nodal points of the structure

F is the vector of the external and control forces acting on the structure

Equation (1) is put in the modal space by using the following weighted modal transformation :

$$\delta = \phi U \quad (2)$$

where U is the modal coordinates of the system

ϕ is the weighted modal shape matrix of the eigenvectors of the flexible system

Using such transformation, reduces the coupled equation of motion (1) to the following uncoupled form :

$$\ddot{U} + \lambda U = f \quad (3)$$

where λ is a diagonal matrix of the eigenvalues of the system

f is the modal force vector given by

$$f = \phi^T F = \begin{bmatrix} \phi_1(l_1) & . & . & . & . & \phi_1(l_N) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \phi_N(l_1) & . & . & . & . & \phi_N(l_N) \end{bmatrix} * F \quad (4)$$

where $\phi_i(l_j)$ is the modal shape at mode i and location l_j .

The modal control forces f , in equation (4), are determined in all the three algorithms from the close form solution of the Riccati Equation such that the control force f_i of the i^{th} mode is :

$$f_i = -(g_1 \omega_1 u_1 + g_2 \dot{u}_1) / R \quad (5)$$

where R is a factor that weighs the importance of minimizing the

vibration with respect to the control forces.

ω_i is the resonant frequency at the i^{th} normal mode.

u_i, \dot{u}_i are the modal displacement and velocity respectively.

g_1, g_2 are the modal position and velocity feedback gains given by [7] as :

$$g_1 = -\omega_i R + ((\omega_i R)^2 + \omega_i^2 R)^{1/2} \quad (6)$$

$$g_2 = \{2R\omega_i[-\omega_i R + ((\omega_i R)^2 + \omega_i^2 R)^{1/2}] + \omega_i^2 R\}^{1/2} \quad (7)$$

Accordingly, the displacement u_i and velocity \dot{u}_i at the i^{th} mode can be feedback and used along with equations (5), (6) and (7) to determine the modal control force f_i .

(I) IMSC METHOD

The IMSC is based primarily on the premise that the number of actuators is equal to the number of controlled modes. Such a premise is attributed to the facts that the physical control forces F , in equation (4) can be determined exactly from the modal forces f by writing :

$$F = (\phi^T)^{-1} f \quad (8)$$

and that the necessary condition for the existence of $(\phi^T)^{-1}$ is the order of actual physical forces F be equal to order of modal forces f . Accordingly, this premise poses serious practical limitations to the application of the IMSC to control large structures since the number of modes retained in the mathematical models may be very large and it is impossible to use an equally large number of actuators to control all

these modes. Therefore, equation (4) can be rewritten as :

$$f = \begin{bmatrix} f_C \\ f_R \end{bmatrix} = \begin{bmatrix} B_{CC} & B_{CR} \\ B_{RC} & B_{RR} \end{bmatrix} \begin{bmatrix} F_C \\ 0 \end{bmatrix} \quad (9)$$

or

$$f_C = B_{CC}F_C \quad (10)$$

$$f_R = B_{RC}F_C \quad (11)$$

where $f_{C,R}$ are the modal forces on the controlled and residual modes respectively.

F_C are the physical forces on the controlled modes.

Once the modal control forces f_C are calculated, equation (10) is solved to give the physical applied forces F_C as :

$$F_C = B_{CC}^{-1}f_C \quad (12)$$

Then equation (11) is used to calculate the modal forces f_R that would excite the residual modes which are generated by the spillover from the controlled modes.

The IMSC method assumes that $f_R=0$ and of course this can only be true if the number of controlled modes is very large compared to the number of residual modes or when the residual modes are at much higher frequency band than the controlled modes.

(II) PI METHOD

The principle limitation of the IMSC method is the requirement that

the number of actuators be equal to the number of controlled modes. When the number of actuators is less than the number of controlled modes, equation (8) will no longer be valid. Consider equation (4) when the control modes are equal to the modelled modes (N) but fewer actuators (M) are to be used, thus it reduces to :

$$f = \begin{bmatrix} f_1 \\ . \\ . \\ f_N \end{bmatrix} = \begin{bmatrix} \phi_1(l_1) & . & . & . & . & \phi_1(l_M) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \phi_N(l_1) & . & . & . & . & \phi_N(l_M) \end{bmatrix} * \begin{bmatrix} F_1 \\ . \\ . \\ F_M \end{bmatrix} \quad (M < N) \quad (13)$$

or

$$f = \varphi^T F \quad (14)$$

Since φ^T is not a square matrix, F cannot be obtained directly through matrix inversion, The solution of equation (14) for F in terms of f can be obtained using least-square approximation, which is :

$$F = B^{-1} f \quad (15)$$

where B^{-1} is the pseudo-inverse of φ^T given by

$$B^{-1} = (\varphi \varphi^T)^{-1} \varphi \quad (16)$$

Once the physical forces are calculated, the actual modal control forces can be computed using equation (13). As noted before, these approximately computed modal forces will not be equal to the optimal modal forces determined from the IMSC algorithm.

(III) MIMSC METHOD

The MIMSC method ranks the modes of vibration according to their modal energy ($\omega_i^2 u^2 + \dot{u}^2$) and dedicates the desired M actuators to control the highest M modes. The modal control forces f_C generated to control these modes are computed from equations (5), (6) and (7). The exact realization F_C of these forces is computed from equation (12) and the spillover into the residual modes is calculated from equation (10).

A flow chart of the MIMSC algorithm is shown in Figure (1). The chart indicates that the time sharing strategy will work first to attenuate the modal energy of the controlled modes. During that time the control spillover will excite the uncontrolled modes. When the modal energy of the uncontrolled modes starts exceeding that of the controlled modes, the actuators are switched to control these high energy modes in order to damp out their vibrations. Such time sharing of the actuators between the modes will eventually bring all these modes under control.

This strategy along with the exact realization of the control forces favor the MIMSC method over the IMSC and PI methods in controlling the vibration of large systems with only a few actuators.

NUMERICAL EXAMPLES

I. MULTI SPRING-MASS SYSTEM

Figure (2) shows a multi spring-mass system which is considered as a

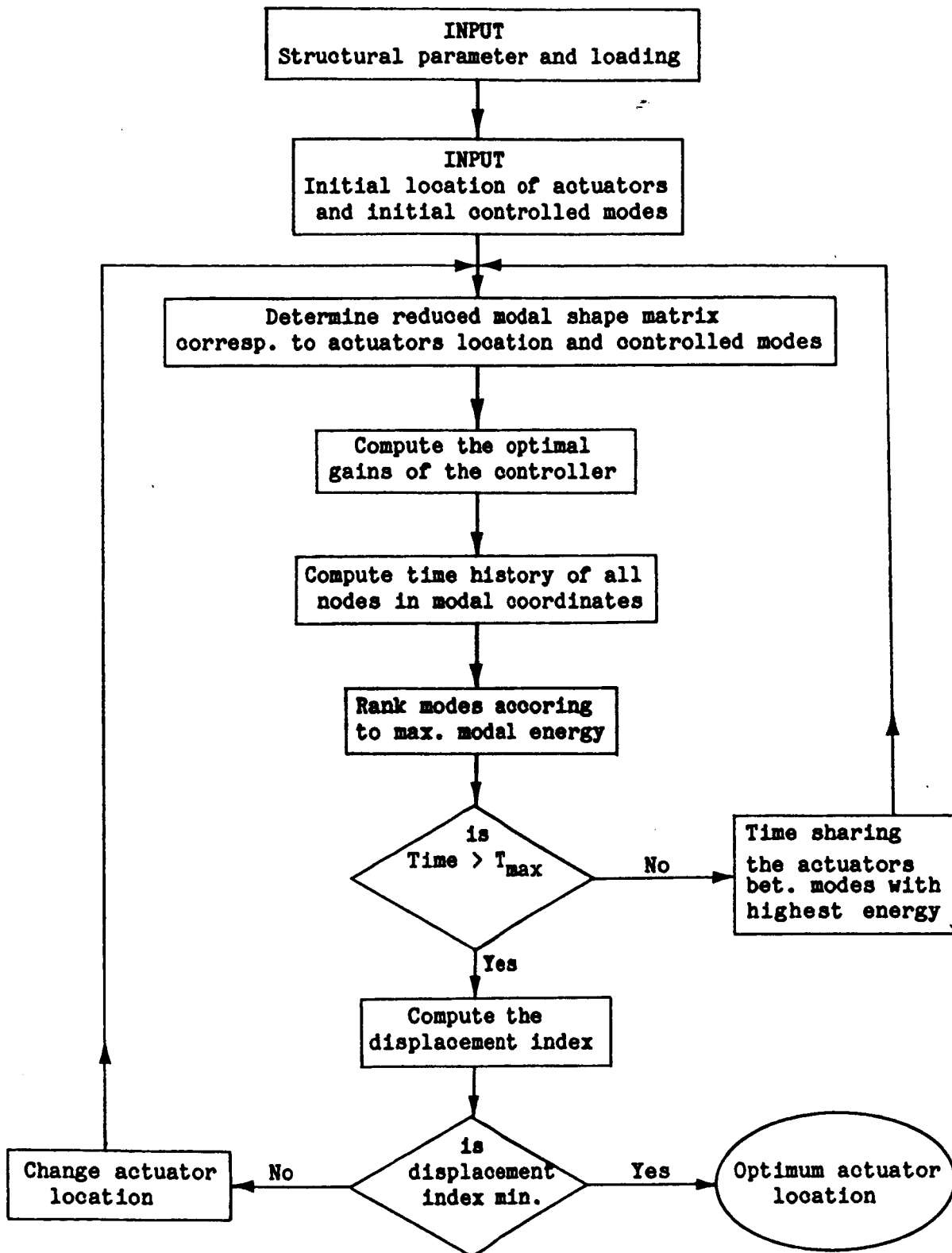


Figure (1) - Flow chart of the MIMSC computational algorithm

$$k_i=1, m_i=1$$

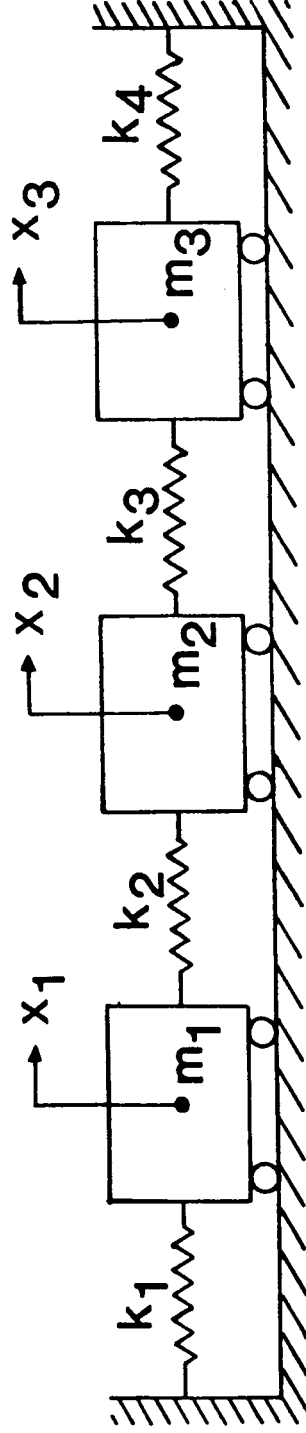


Figure (2) - A spring-mass system

simple example of a flexible system to illustrate the characteristic of the three algorithms. The main dynamic characteristics of this system are given in Table (1).

(i) Using IMSC method

The three masses of the flexible system shown in Figure (2) are displaced initially 1, -1 and 0 respectively from their equilibrium positions and then left to vibrate under the action of an IMSC controller with all the states are observed. The controller is designed to control the first mode of vibration through the use of one actuator placed at the first mass.

Figure (3-a) shows the time history of the amplitudes of vibration of the three masses. The figure indicates that the IMSC method failed, as predicted, to control all the 3 modelled modes of vibration of the system. Such drawback can be related directly to the fact that the actuator has been utilized only to eliminate the first mode and no control action is provided to the residual two modes.

(ii) Using PI method

Figure (3-b) shows the time history of the amplitudes of vibration of three masses as obtained by the PI method with one actuator placed at the first mass.

Although the modal control forces are obtained by using a least-

Table (1) - Dynamic characteristics of a three spring-mass system.

Stiffness Matrix		
2.0000	-1.0000	0.0000
-1.0000	2.0000	-1.0000
0.0000	-1.0000	2.0000
Mass Matrix		
1.0000	0.0000	0.0000
0.0000	1.0000	0.0000
0.0000	0.0000	1.0000
Eigenvalues		
0.5858	2.0000	3.4142
Eigenvectors		
0.5000	-0.7071	0.5000
0.7071	0.0000	-0.7071
0.5000	0.7071	0.5000

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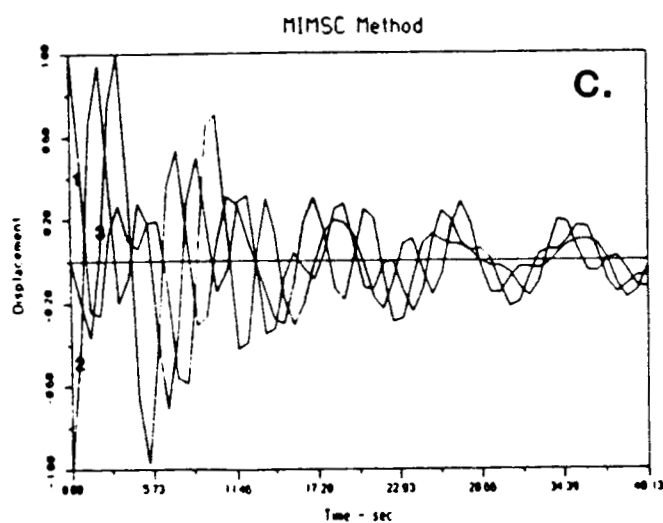
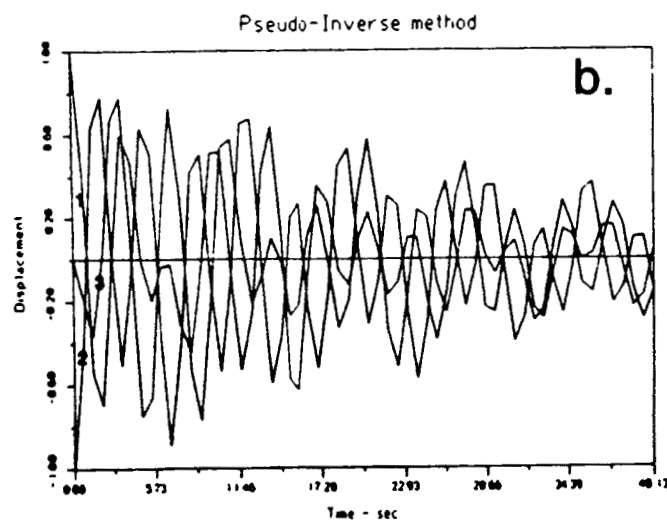
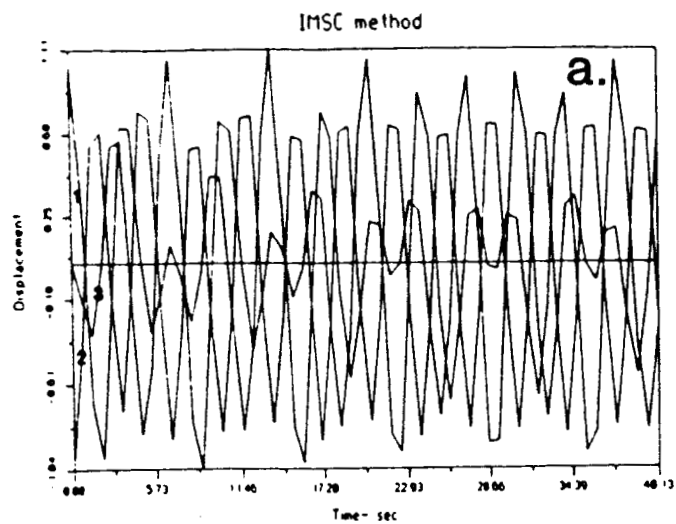


Figure (3) - Time histories of amplitudes of vibration of spring-mass system using IMSC, PI and MIMSC methods with one actuator ($R=100$)

square approximation, the figure indicates that this method can result in damping out the vibrations of the system. This is due to the fact that all the modes are controlled but by virtue of the in-exact nature of the feedback control law, the process of suppression of the vibration is not as dramatic as expected.

(iii) Using MIMSC method

With the time sharing concept, the MIMSC utilizes effectively the installed actuators such that these actuators will provided exact control action to the dominant modes and sequence the controller among all the modes until the vibrations of the system is completely damped out. Accordingly, in the considered example, the actuator is powered by signals to eliminate all the three modes of the system and not only the first mode as in the IMSC. This is achieved by time sharing the actuator, among the three modes, based on the maximum modal energy ranking.

Figure (3-c) shows the time history of the amplitudes of vibration of the three masses when the MIMSC method is utilized.

The figure indicates that sharing small number of actuators among a larger number of modes has been effective in damping out quickly the amplitudes of vibration of all the modes.

The effectiveness of the MIMSC can best be understood by considering Figures(4-a) and (4-b).

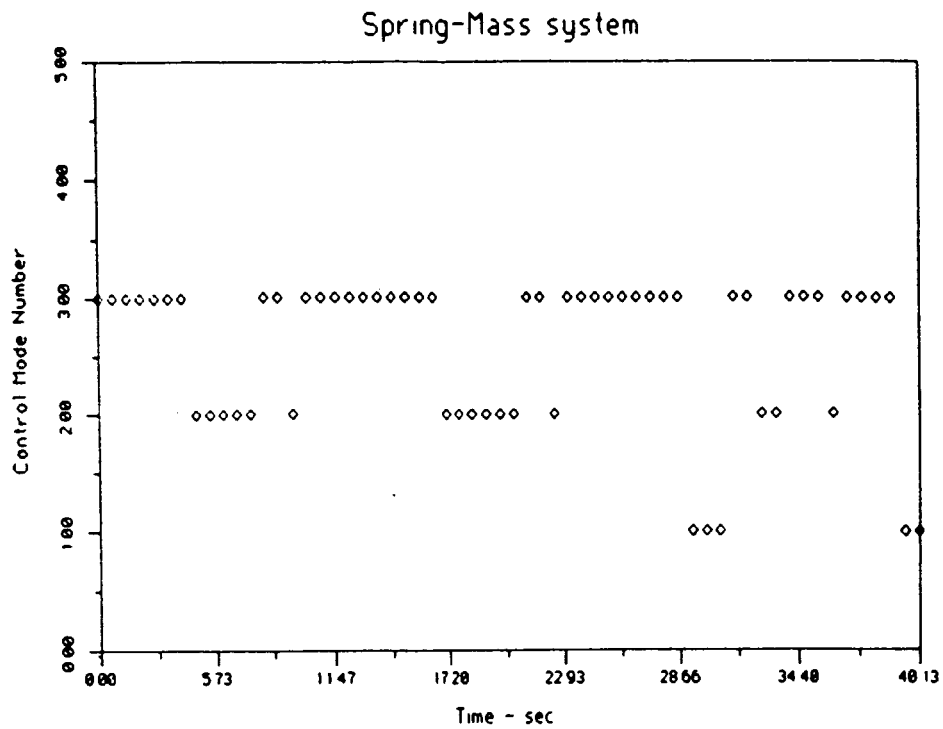


Figure (4-a) - Control mode of highest modal energy for the spring-mass system

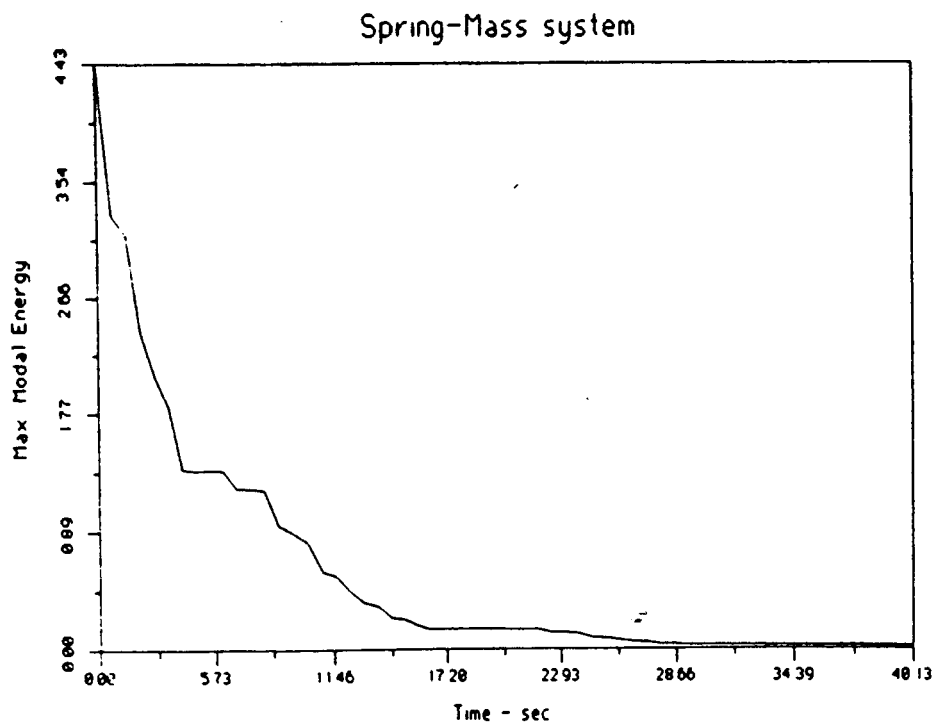


Figure (4-b) - Maximum modal energy of the spring-mass system

Figure (4-a) shows the control that has the highest modal energy at any instant of time. It can be seen that the actuator is used first to attenuate the amplitude of vibration of the third mode, which currently has the highest modal energy. After a small time interval the control action is switched to control the second mode since its modal energy becomes higher than the energy of the first and the third modes. This action of time sharing the single actuator between the modes continues until all three modes are brought under control. This is demonstrated clearly in Figure (4-b) by the continuously decaying vibration energy of the system.

A better quantitative comparison between the three methods can be established based on the displacement index U_d which is given by :

$$U_d = \sum_{t=0}^{t=t^*} \sum_{i=1}^N \delta_i^2 \Delta t \quad (17)$$

where N is the number of d.o.f. of system

Δt is the integration time increment

t^* is the maximum time limit of integration

Table (2) summarizes the results of such a comparison when one actuator is placed at mass 1 as well as when two actuators are used at masses 1 and 2.

The table indicates clearly that the MIMSC is very effective in damping out the vibration of the 3-mass system particularly when very small number actuators are used as compared to the IMSC and PI

Table (2) - Effect of the control algorithm on the displacement index for spring-mass system when one or two actuators are used with $R=100$.

method	Using 1 actuator	Using 2 actuators
IMSC	39.47	29.21
PI	14.42	6.46
MIMSC	9.07	5.06

algorithms.

II. CANTILEVER BEAM

Figure (5) shows a flexible steel cantilever beam modeled by a 9-finite element model that has 9 d.o.f. of linear translations and 9 d.o.f. of angular rotations with node 1 fixed. The beam is 0.45m long and has rectangular cross section which is 0.0125m wide and 0.0021m thick. For this beam, the lowest natural frequency is found to be 8.6Hz. In this example, the beam is subjected to a impulsive load of magnitude 1.0N for 1.0ms at the free end.

(a) Control by one linear actuator

A force actuator, placed at the tip of the flexible beam, is used to control the vibration of this beam model.

Figures (6-a), (6-b) and (6-c) show the time histories of the amplitudes of transverse vibrations of the beam at its free end as obtained by the application of IMSC, PI and MIMSC algorithms respectively.

The figures indicate that the IMSC is again successful in suppressing the lowest mode of vibration but all the higher modes remain totally undamped. On the contrary, the PI and MIMSC methods exhibit complete control over all the modes. However, the response of the system

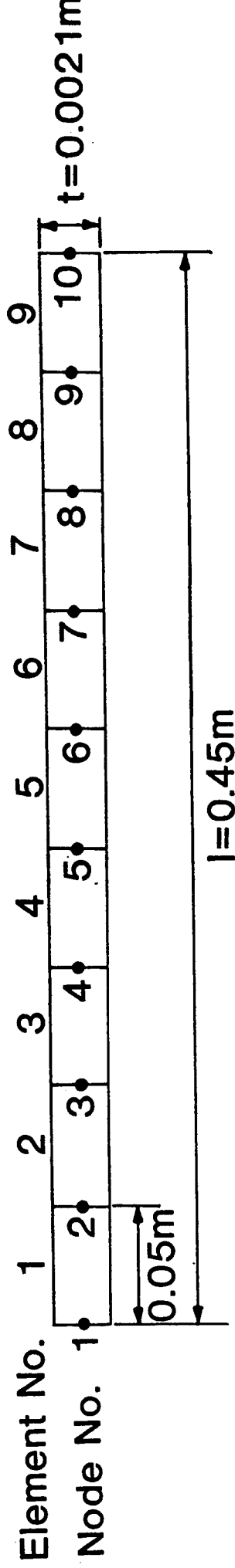


Figure (5) - Layout of a 9-element straight beam

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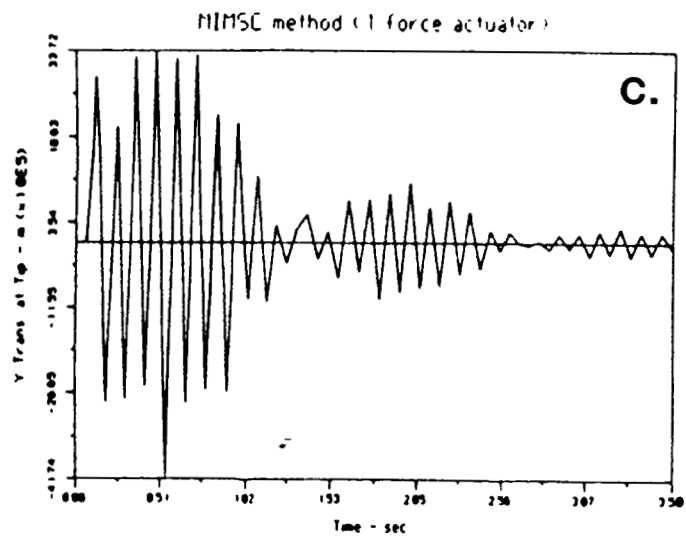
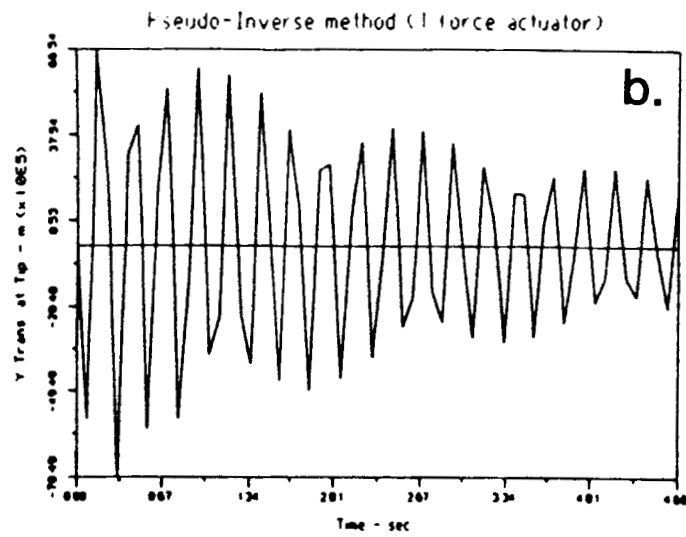
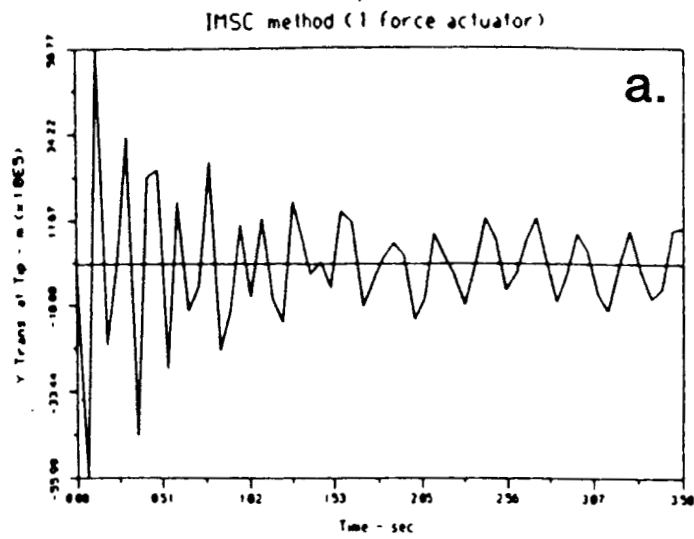


Figure (6) - Time history of the amplitudes of transverse vibration of cantilever beam using IMSC, PI and MIMSC methods with one linear actuator ($R=1000$)

when using the MIMSC method indicates faster decay of the vibration than the PI method. Furthermore, with the MIMSC method, the maximum amplitude of oscillation of the beam is about 68.1% and 28.8% lower than those obtained by the PI method and the IMSC method respectively.

Figures (7-a) and (7-b) show the mode of the highest modal energy at any time as well as the instantaneous modal energy of the beam respectively when the control action is based on the MIMSC method.

(b) Control by one torque actuator

When a single torque actuator, placed at element 1, is used to control the vibration of the flexible cantilever beam, then the resulting time histories of the amplitudes of transverse vibration of the beam tip are as shown in Figure (8-a), (8-b) and (8-c) for IMSC, PI and MIMSC controllers respectively.

Again, the figures emphasize that the MIMSC method damp out all the vibration modes effectively. But, in PI method, the beam continue to vibrate without decaying with an amplitude higher than those in IMSC and MIMSC methods. This is because the errors created by the least-square approximation of 18 modal forces by one physical force is very high that renders the feedback control law ineffective.

Quantitatively, the comparison of the three method when a force or torque actuators are used is given in Table (3).

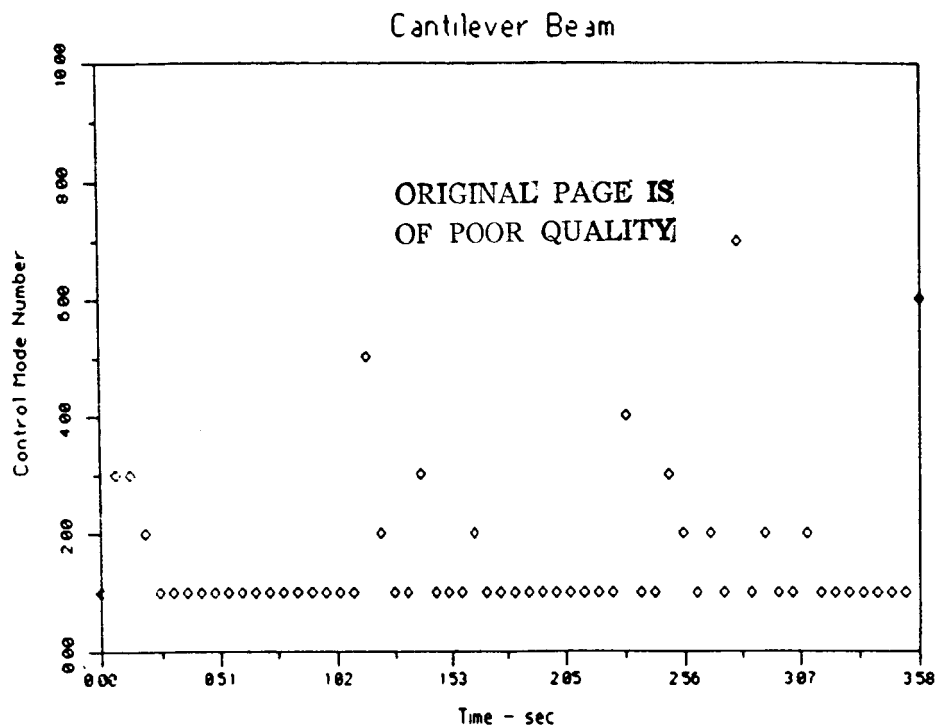


Figure (7-a) - Control mode of highest modal energy for the cantilever beam

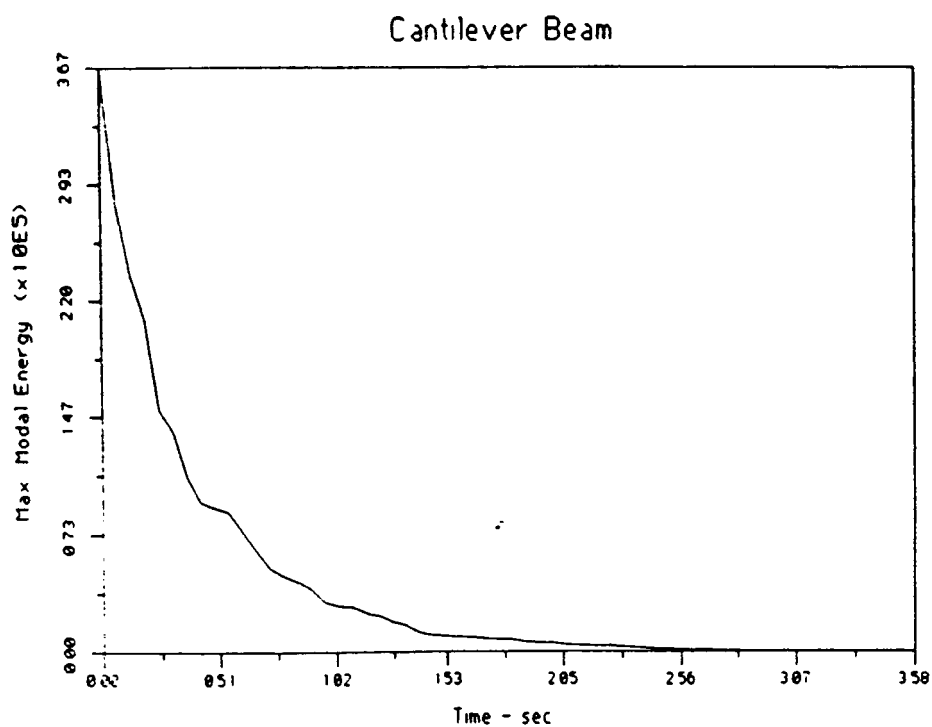


Figure (7-b) - Maximum modal energy of cantilever beam

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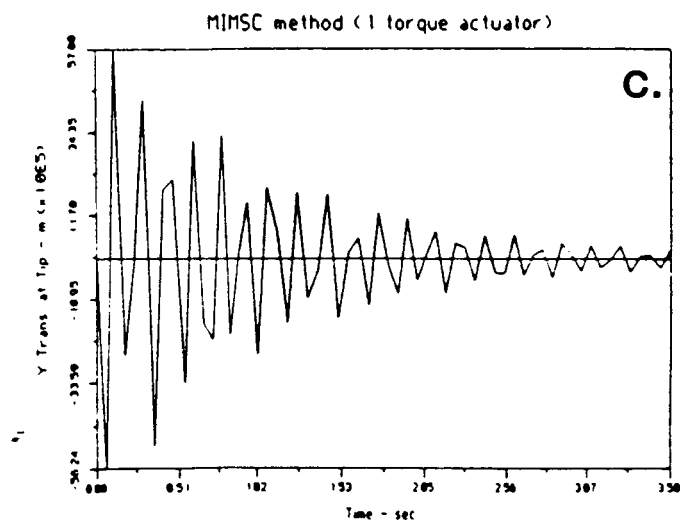
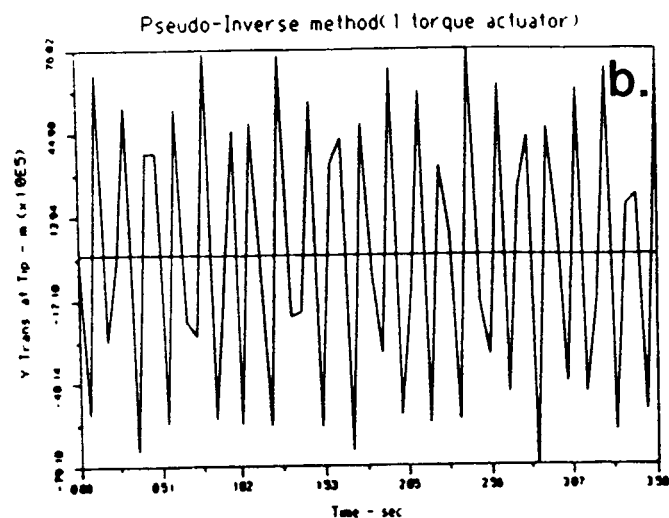
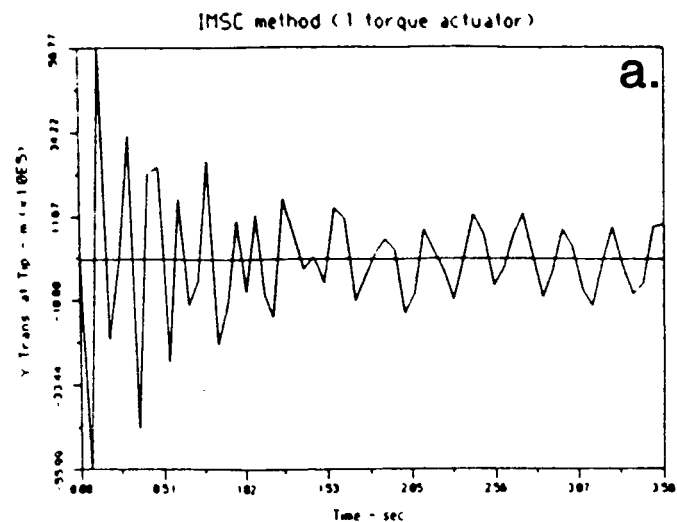


Figure (8) - Time history of the amplitudes of transverse vibration of cantilever beam using IMSC, PI and MIMSC methods with one torque actuator ($R=1000$)

Table (3) - Effect of the control algorithm on the displacement index for cantilever beam when using one linear or rotary actuator with $R=1000$.

method	Using force actuator ($\times 10^6$)	Using torque actuator ($\times 10^6$)
IMSC	19.64	17.25
PI	29.56	68.12
MIMSC	6.70	9.78

CONCLUSIONS

This paper has presented a comparative study between three active control algorithms, IMSC, PI and MIMSC, which are suitable for controlling large flexible systems.

The study showed that when small number of actuators are used to control large number of modes then the MIMSC results in faster and effective suppression of the vibrations. The IMSC method is found, however, to be effective in damping out the amplitudes of low frequency modes but due to the fact that the IMSC does not account for control spillover it is demonstrated here that the high frequency modes remain uncontrolled.

With the PI method it is shown that the in-exact realization of the modal controller can result in slower damping of the vibration when the number of actuators (M) is not far smaller than the number of controlled modes (N). But when $M \ll N$ the least square nature of the PI would result in degradation of the controller performance as demonstrated clearly in the case of the cantilever beam.

The study emphasized the potential of the MIMSC for being a viable and efficient method for controlling the vibrations of large systems with only few actuators.

ACKNOWLEDGEMENTS

Special thanks are due to the Space Science and Technology division at NASA Goddard Space Flight Center for providing the funds necessary to conduct this study under Grants NAG 5-520 and NAG 5-749.

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